

The Evolution of Mathematics: From Vedic Methods to Modern Mathematics

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ABSTRACT

Mathematics is a science that involves the study of numbers. There is often a comparison between two types of mathematics: Vedic and modern. In this paper, we will explore both methods to determine which is more effective. In modern mathematics, when calculations become lengthy, we often rely on calculators. However, Vedic mathematics allows us to solve even complex calculations in less time compared to modern methods. The results of Vedic mathematics are the outcome of continuous practice by Swami Bharati Krishna Tirthaji Maharaj. Vedic mathematics uses 16 sutras and 13 sub-sutras to solve various problems. In this paper, we will use Meru-Prastara to find binomial expansions and perform differentiation.

Keywords: Binomial Theorem, Differentiation & Integration, Meru-Prastar, Pascal Triangle, Vedic Mathematics

A special method for finding the number of combinations, called Meru-Prastara, is described in Chandah Sutras (200 BC). It is basically the same triangular array commonly known as Pascal's triangle. Pascal's Triangle is a triangular array of numbers where each entry is the sum of the two numbers directly above it. It starts with a single "1" at the top. Each row corresponds to the coefficients in the expansion of a binomial expression $((a + b)^n)$. Pascal triangle is named after French mathematician Blaise Pascal. Vedic mathematics, the formula for removing double multiples was given only in the third century. Which was called Meru-Prastar, where 'Meru' i.e. mountain and 'prastar' i.e. ladder of mountain. The first description of the 6 lines of MeruPrastaris in Pingal's verse weapons. The 'merupstar' describing the difference of verses is comparable to the triangle of pascal. The meruprastara rule by Pingal is explained by Halayudh in his mrintasajivani as follows.

History of the Binomial Theorem-

The history of the Binomial theorem is very entertaining. It is often believed that Pascal did the work of desiring the Binomial multiplication as a triangle, but pingle, the third-century Indian mathematician, has used the biverse multiplication beautifully in verses: sutram. This was called 'Meruprastara'. According to Janshruti, it was the anuj of Panini, He mentions the meruprastara, the biverse theorem and the double-edged number; the stalled chanuimatdha in the verse sutras.

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APPLICATION OF MERU-PRASTAR:

Meru Prastar, also known as the Pascal's Triangle in Vedic mathematics, is a triangular arrangement of numbers where each number is the sum of the two directly above it. It's used to find combinations and coefficients in binomial expansions. In Vedic texts, it represents various combinatorial and algebraic concepts. Each row corresponds to the coefficients of the binomial expansion of $((a + b)^n)$.

Pascal's Triangle

						1					
					1			1			
				1		2		1			
		1		3		3		1			
	1		4		6		4		1		
1		5		10		10		5		1	

MULTIPLICATION BY VEDIC METHOD:

Example: Find the value of $(1002)^5$

Solution: - $a = 1, b = 002$

$$\begin{aligned}
 (a+b)^5 &= a^5 | 5a^4b^1 | 10a^3b^2 | 10a^2b^3 | 5ab^4 | a^0b^5 \\
 &= (1)^5 | 5(1)^4(002) | 10(1)^3(002)^2 | 10(1)^2(002)^3 | 5(1)(002)^4 | (002)^5 \\
 &= 1 | 5 \times 2 | 10 \times 4 | 10 \times 8 | 5 \times 16 | 32 \\
 &= 1 | 010 | 040 | 080 | 080 | 32 \\
 &= 1 \ 010 \ 040 \ 080 \ 080 \ 32
 \end{aligned}$$

MULTIPLICATION BY MODERN METHOD:

Example: Find the value of $(1002)^5$

Solution: - General Formula

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots + {}^nC_n a^0 b^n$$

According to the question, on writing 5 in place of n in the formula, because the power of the question is 5, $(a+b)^5 = {}^5C_0 a^5 b^0 + {}^5C_1 a^4 b^1 + {}^5C_2 a^3 b^2 + {}^5C_3 a^2 b^3 + {}^5C_4 a^1 b^4 + {}^5C_5 a^0 b^5$

$$\begin{aligned}
 (1000+2)^5 &= {}^5C_0 (1000)^5 (2)^0 + {}^5C_1 (1000)^4 (2)^1 + {}^5C_2 (1000)^3 (2)^2 + {}^5C_3 (1000)^2 (2)^3 + {}^5C_4 (1000)^1 (2)^4 + {}^5C_5 (1000)^0 (2)^5 \\
 &= 1 \times 1000000000000000 \times 1 + 5 \times 100000000000000 \times 2 + 10 \times 10000000000 \times 4 + 10 \times 1000000 \times 8 \\
 &\quad + 5 \times 1000 \times 16 + 1 \times 32 \\
 &= 1000000000000000 + 10000000000000 + 40000000000 + 80000000 + 80000 + 32 \\
 &= 1 \ 010 \ 040 \ 080 \ 080 \ 32
 \end{aligned}$$

DIFFERENTIATION BY VEDIC METHOD:

For higher derivative like D_2, D_3, D_4, D_5 the Vedic sub formula “proportionately” is applied. Along with this to find the coefficients we use Meru Prastar method also known as Pascal's triangle.

Coefficients	Derivatives
1	D (given function)
1, 1	D ₁ (First derivative)
1, 2, 1	D ₂ (Second derivative)
1, 3, 3, 1	D ₃ (Third derivative)
1, 4, 6, 4, 1	D ₄ (Fourth derivative)
1, 5, 10, 10, 5, 1	D ₅ (Fifth derivative)

Example: Find the third derivative of $x^4 e^{2x}$

Solution: - Since the coefficient for power 3 will be 1, 3, 3, 1 then

$$a = x^4, b = e^{2x}$$

$$\frac{d^3}{dx^3} (x^4 e^{2x})$$

$$= \frac{d^0}{dx^0} (x^4) \frac{d^3}{dx^3} (e^{2x}) \left| 3 \frac{d^1}{dx^1} (x^4) \frac{d^2}{dx^2} (e^{2x}) \right| 3 \frac{d^2}{dx^2} (x^4) \frac{d^1}{dx^1} (e^{2x}) \left| \frac{d^3}{dx^3} (x^4) \frac{d^0}{dx^0} (e^{2x}) \right|$$

$$\frac{d^3}{dx^3} (x^4 e^{2x}) = x^4 \cdot 2 \cdot 2 \cdot 2 e^{2x} | 3 \cdot 4x^3 \cdot 2 \cdot 2 \cdot e^{2x} | 3 \cdot 4 \cdot 3x^2 \cdot 2 \cdot e^{2x} | 4 \cdot 3 \cdot 2 \cdot 2x e^{2x}$$

$$\frac{d^3}{dx^3} (x^4 e^{2x}) = 8x^4 e^{2x} + 48x^3 e^{2x} + 72x^2 e^{2x} + 48x e^{2x}$$

DIFFERENTIATION BY VEDIC METHOD:

For higher derivative like D, D₂, D₃, D₄ the Vedic sub formula Urdhava-triayagbhyam which means “Vertically and crosswise” is applied. Along with this to find the coefficients we use Meru Prastar method also known as Pascal’s triangle. (For 2nd derivative)

	D	D ₁	D ₂
	*	*	*
	*	*	*
Coefficients	1	2	1

Example: Find the second derivative of $x^4 e^{3x}$.

Solution: - Since the coefficient for power 3 will be 1, 2, 1

Then,

	D	D ₁	D ₂
	x^4	$4x^3$	$12x^2$
	e^{3x}	$*3e^{3x}$	$*9e^{3x}$
Coefficients	1	2	1

$$\frac{d^2}{dx^2} (x^4 e^{3x}) = 9x^4 e^{3x} + 24x^3 e^{3x} + 12x^2 e^{3x}$$

Example: Find the third derivative of $x^4 e^{4x}$.

Solution: - Since the coefficient for power 3 will be 1, 3, 3, 1.

	D	D ₁	D ₂	D ₃
	*	*	*	*
	*	*	*	*
Coefficients	1	3	3	1

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	D	D ₁	D ₂	D ₃
	x^4	$4x^3$	$12x^2$	$24x$
	$*e^{3x}$	$3e^{3x}$	$9e^{3x}$	$27e^{3x}$
Coefficients	1	3	3	1

$$\frac{d^3}{dx^3}(x^4 e^{3x}) = x^4 27e^{3x} + 3 \times 4x^3 9e^{3x} + 3 \times 3e^{3x} 12x^2 + e^{3x} 24x$$

$$\frac{d^3}{dx^3}(x^4 e^{3x}) = (27x^4 + 108x^3 + 108x^2 + 24x) e^{3x}$$

DIFFERENTIATION BY MODERN METHOD:

Example: Find the third derivative of $x^4 e^{3x}$

Solution:

Differentiate with respect to x

$$\frac{d}{dx}(x^4 e^{3x}) = x^4 \frac{d}{dx}(e^{3x}) + e^{3x} \frac{d}{dx}(x^4)$$

$$\frac{d}{dx}(x^4 e^{3x}) = 3x^4 e^{3x} + 4e^{3x} x^3$$

$$\frac{d}{dx}(x^4 e^{3x}) = (3x^4 + 4x^3) e^{3x}$$

Differentiate with respect to x

$$\frac{d^2}{dx^2}(x^4 e^{3x}) = (3x^4 + 4x^3) \frac{d}{dx}(e^{3x}) + e^{3x} \frac{d}{dx}(3x^4 + 4x^3)$$

$$\frac{d^2}{dx^2}(x^4 e^{3x}) = (3x^4 + 4x^3) 3e^{3x} + e^{3x} (12x^3 + 12x^2)$$

$$\frac{d^2}{dx^2}(x^4 e^{3x}) = (9x^4 + 24x^3 + 12x^2) e^{3x}$$

Differentiate with respect to x

$$\frac{d^3}{dx^3}(x^4 e^{3x}) = (9x^4 + 24x^3 + 12x^2) \frac{d}{dx}(e^{3x}) + e^{3x} \frac{d}{dx}(9x^4 + 24x^3 + 12x^2)$$

$$\frac{d^3}{dx^3}(x^4 e^{3x}) = (27x^4 + 72x^3 + 36x^2) e^{3x} + (36x^3 + 72x^2 + 24x) e^{3x}$$

$$\frac{d^3}{dx^3}(x^4 e^{3x}) = (27x^4 + 108x^3 + 108x^2 + 24x) e^{3x}$$

INTEGRATION-

Integration is the calculation of Integral, Integration is used to find many useful quantities such as areas, volumes, displacement etc. Integration is a method of adding or summing up the parts to find the whole. It is reverse process of differentiation.

INTEGRATION BY VEDIC METHOD:

Integration is just reverse of differentiation. For differentiation we use Ekanyunena-Purvena (means one less than the previous one) and integration we use Ekadhiken-Purvena (means one more than the previous one) for the power function.

In general form:

Of $\int f_1 \cdot f_2 dx$
 Differentiate $f_1 \dots f_1$
 Integrate $f_2 \dots f_2$
 So $\int f_1 \cdot f_2' - \int f_2' \cdot f_1 dx$

Here, of the two functions write the one which gets differentiated to zero and write its differential in front of as follows. Write the second function below the first followed by its integral value and apply Urdhava- triyagbhyam which means “Vertically and crosswise” method as follows. The sign of the product are alternately +ve & -ve. Now we can do the final integration.

In general form:

f(x)							
Differentiate	f	f'	f''	f'''	f''''	...to 0	...
Integrate	$\int f$	$\int f dx$	$\int \int f dx$	$\int \int \int f dx$	$\int \int \int \int f dx$...	
		+	-	+	-

Example: Find the Integration of $x^3 e^{4x}$

Solution: -

$x^3 e^{4x}$					
Differentiate	x^3	$3x^2$	$6x$	6	0
Integrate	e^{4x}	$\frac{1}{4} e^{4x}$	$\frac{1}{16} e^{4x}$	$\frac{1}{64} e^{4x}$	$\frac{1}{256} e^{4x}$
		+	-	+	-

Then,

$$\int x^3 e^{4x} dx = x^3 \frac{e^{4x}}{4} - 3x^2 \frac{e^{4x}}{16} + 6x \frac{e^{4x}}{64} - 6 \frac{e^{4x}}{256}$$

$$\int x^3 e^{4x} dx = \left(\frac{x^3}{4} - \frac{3x^2}{16} + \frac{6x}{64} - \frac{6}{256} \right) e^{4x}$$

INTERGRATION BY MODERN METHOD:

Example: Find the Integration of $x^3 e^{4x}$

Solution: -

Applying Ist - IInd Rule-

$$\int x^3 e^{4x} dx = x^3 \int e^{4x} dx - \int \left(\frac{d}{dx} x^3 \int e^{4x} dx \right) dx$$

$$\int x^3 e^{4x} dx = x^3 \frac{e^{4x}}{4} - \int \left(\frac{3}{4} x^2 e^{4x} \right) dx$$

$$\int x^3 e^{4x} dx = x^3 \frac{e^{4x}}{4} - \frac{3}{4} \int x^2 e^{4x} dx$$

Again, Applying Ist - IInd Rule-

$$\int x^3 e^{4x} dx = x^3 \frac{e^{4x}}{4} - \frac{3}{4} \left[x^2 \int e^{4x} dx - \int \left(\frac{d}{dx} x^2 \int e^{4x} dx \right) dx \right]$$

$$\int x^3 e^{4x} dx = x^3 \frac{e^{4x}}{4} - \frac{3}{4} \left[x^2 \frac{e^{4x}}{4} - \int \left(2x \frac{e^{4x}}{4} \right) dx \right]$$

$$\int x^3 e^{4x} dx = x^{3x} \frac{e^{4x}}{4} - \frac{3}{16} x^2 e^{4x} + \int \frac{6}{16} x e^{4x} dx$$

$$\int x^3 e^{4x} dx = x^{3x} \frac{e^{4x}}{4} - \frac{3}{16} x^2 e^{4x} + \frac{6}{16} \int x e^{4x} dx$$

Again, Applying Ist - IInd Rule-

$$\int x^3 e^{4x} dx = x^{3x} \frac{e^{4x}}{4} - \frac{3}{16} x^2 e^{4x} + \frac{6}{16} \left[x \int e^{4x} dx - \int \left(\frac{d}{dx} x \int e^{4x} dx \right) dx \right]$$

$$\int x^3 e^{4x} dx = x^{3x} \frac{e^{4x}}{4} - \frac{3}{16} x^2 e^{4x} + \frac{6}{16} \left[x \frac{e^{4x}}{4} - \int \frac{e^{4x}}{4} dx \right]$$

$$\int x^3 e^{4x} dx = x^{3x} \frac{e^{4x}}{4} - \frac{3}{16} x^2 e^{4x} + \frac{6}{64} x e^{4x} - \frac{6}{256} e^{4x}$$

$$\int x^3 e^{4x} dx = \left(\frac{x^3}{4} - \frac{3x^2}{16} + \frac{6x}{64} - \frac{6}{256} \right) e^{4x}$$

CONCLUSION

The 16 sutras and 13 sub-sutras of Vedic mathematics were derived from the Vedas, Vedanta, Vedic literature, and the Puranas. These sutras are simple and easy to apply. In modern mathematics, there is virtually no area where they are not used. They are applied in arithmetic, algebra, geometry, trigonometry, and even in calculus. Swami Bharati Krishna Tirthaji composed a series of 16 works that provide detailed explanations of these Vedic sutras. Vedic mathematics enables quicker solutions to complex problems compared to modern methods.

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Conflict of Interest

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